WEEKLY TEST TYJ TEST - 29 B
SOLUTION Date 01-12-2019

[PHYSICS]

1. **(c)** $W_{AB} = -\left(P_0V_0 + \frac{P_0V_0}{2}\right) = -\frac{3}{2}P_0V_0$

$$W_{BC} = (2P_0)(2V_0) + \frac{P_0(2V_0)}{2} = 5P_0V_0$$

$$W_{ABC} = \frac{7}{2}P_0V_0$$

2. **(a)** $\Delta E_{int} = 0$, for a complete cycle and for given cycle work done is negative, so from first law of thermodynamics Q will be negative i.e., $Q < 0$.

3. **(d)** $PV^m =$ const.

$$\Rightarrow \quad V^m dP + mv^{m-1}\,Pd\,v = 0$$

$$\Rightarrow \quad \frac{dP}{dV} = \frac{-mP}{V} = \tan(180 - 37°)$$

$$\Rightarrow \quad \frac{3}{4} = m\frac{2x10^5}{4x10^5} \Rightarrow m = \frac{3}{2}$$

4. **(b)** $\because \eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} T_2 = 300\text{ K}$

where Q_1 = heat absorbed, Q_2 = heat rejected

$$\Rightarrow \quad 1 - \frac{T/3}{T} = \frac{W}{Q_1} \quad \Rightarrow \quad \frac{2}{3} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\Rightarrow \quad \frac{2}{3} = 1 - \frac{Q_2}{Q_1} \Rightarrow \frac{Q_2}{Q_1} = \frac{1}{3} \quad \Rightarrow \quad Q_2 = \frac{Q_1}{3} = \frac{Q}{3}$$

5. **(b)** In first case $\eta_1 = \frac{T_1 - T_2}{T_1}$

In second case $\eta_2 = \frac{2T_1 - 2T_2}{2T_1} = \frac{T_1 - T_2}{T_1} = \eta$

6. **(c)** Coefficient of performance

$$K = \frac{T_2}{T_1 - T_2} \Rightarrow 5 = \frac{(273-13)}{T_1 - (273-13)} = \frac{260}{T_1 - 260}$$

$$\Rightarrow \quad 5T_1 - 1300 = 260 \Rightarrow 5T_1 = 1560$$

$$\Rightarrow \quad T_1 = 312\text{ K} \to 39°\text{C}$$

7. **(c)** $\eta = \frac{T_1 - T_2}{T_1} = \frac{W}{Q} \Rightarrow W = \frac{Q(T_1 - T_2)}{T_1}$

$= \frac{6 \times 10^4 [(227 + 273) - (273 + 127)]}{(227 + 273)}$

$= \frac{6 \times 10^4 \times 100}{500} = 1.2 \times 10^4 \text{ cal}$

8. **(d)** Initially $\eta = \frac{T_1 - T_2}{T_1} \Rightarrow 0.5 = \frac{T_1 - (273 + 7)}{T_1}$

$\Rightarrow \frac{1}{2} = \frac{T_1 - 280}{T_1} \Rightarrow T_1 = 560 \text{ K}$

Finally

$\eta_1' = \frac{T_1' - T_2}{T_1'} \Rightarrow 0.7 = \frac{T_1' - (273 + 7)}{T_1'}$

$\Rightarrow T_1' = 933 \text{ K}$

$\therefore$ Increase in temperature

$= 933 - 560 = 373 \text{ K} = 380 \text{ K}$

9. **(a)** $\eta = \frac{T_1 - T_2}{T_1} = \frac{1}{6}$ (i)

$\eta' = \frac{T_1 - (T_2 - 65)}{T_1} = \frac{1}{3}$ (ii)

From equations (i) and (ii)

$\frac{\eta'}{\eta} = \left(\frac{T_1 - T_2 + 65}{T_1}\right)\left(\frac{T_1}{T_1 - T_2}\right) = \frac{(1/3)}{(1/6)} = 2$

or $\frac{T_1 - T_2 + 65}{T_1 - T_2} = 2$

or $T_1 - T_2 = 65$ (iii)

From equation (i), $\frac{65}{T_1} = \frac{1}{6}$ or $T_1 = 390 \text{ K}$

and from equation (iii), $T_2 = T_1 - 65 = 325 \text{ K}$

10. The rate of cooling decreases with the decrease in temperature difference between the body and surroundings.

11. According to Wien's law the wavelength (λ_m) corresponding to which energy emitted per sec per area by a perfectly black body is maximum, is inversely proportional to the absolute temperature (T) of the black body. Temperature of the sun would be maximum of the given three.
As $\lambda_m T = \text{constant}$

12. (c) $W_{AB} = -P_0V_0$, $W_{BC} = 0$ and $W_{CD} = 4P_0V_0$

$\Rightarrow W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

13. (b) W_{AB} is negative (volume is decreasing) and
W_{BC} is positive (volume is increasing) and
since, $|W_{BC}| > |W_{AB}|$

$\therefore$ net work done is positive and area between semicircle which is equal to $\frac{\pi}{2} atm - lt.$

14. (c) $\Delta Q = \mu C_P \Delta T = \frac{7}{2}\mu R \Delta T \quad \left(C_P = \frac{7}{2}R\right)$

$\Delta U = \mu C_V \Delta T = \frac{5}{2}\mu R \Delta T \quad \left(C_V = \frac{5}{2}R\right)$

and $\Delta W = \Delta Q - \Delta U = \mu R \Delta T$

$\Rightarrow \Delta Q : \Delta U : \Delta W = 7 : 5 : 2$

15. C_v for hydrogen $= \frac{5R}{2}$, C_0 for helium $= \frac{3R}{2}$

C_v for water vapour $= \frac{6R}{2}$

$$\therefore \quad [C_v]_{mix} = \frac{4 \times \frac{5R}{2} + 2 \times \frac{3R}{2} + 1 \times 3R}{4 + 2 + 1} = \frac{16R}{7}$$

$$\therefore \quad C_p = C_v + R = \frac{16R}{7} + R = \frac{23R}{7}$$

16. (c) $(C_V)_{mix} = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2} = \frac{1 \times \frac{3}{2}R + 1 \times \frac{5}{2}R}{1+1} = 2R$

$\left((C_V)_{mono} = \frac{3}{2}R,\ (C_V)_{di} = \frac{5}{2}R\right)$

17. Total energy radiated from a body,

$$Q = A\varepsilon\sigma T^4 t$$

$\Rightarrow \quad Q \propto AT^4 \propto r^2 T^4 \qquad (\because A = 4\pi r^2)$

$$\Rightarrow \frac{Q_P}{Q_Q} = \left(\frac{r_P}{r_Q}\right)^2 \left(\frac{T_P}{T_Q}\right)^4 = \left(\frac{8}{2}\right)^2 \left[\frac{(273+127)}{(273+527)}\right]^4 = 1$$

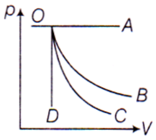
18. (a) p-V graph is not rectangular hyperbola. Therefore, process $A \rightarrow B$ is not isothermal.

(b) In process BCD, product of pV (therefore temperature and internal energy) is decreasing. Further, volume is decreasing. Hence, work done is also negative. Hence, Q will be negative or heat will flow out of the gas.

(c) W_{ABC} = positive

(d) For clockwise cycle on p-V diagram with p on y-axis, net work done is positive.

19. (a) Curve OA represents isobaric process (since pressure is constant). Since, the slope of adiabatic process is more steeper than the isothermal process.



(b) Curve OB represents isothermal process.

(c) Curve OC represents adiabatic process.

(d) Curve OD represents isochoric process (since volume is constant)

20. No change in the internal energy of an ideal gas, but for real gas internal energy increases, because work is done against intermolecular forces.

[CHEMISTRY]

21.

NH_3 donates pair of electrons while BF_3, Cu^{2+} and $AlCl_3$ accept lone pair of electrons.

22.

$$\text{Acid} \xrightarrow{-H^+} \text{Conjugate base}, \quad \text{Base} \xrightarrow{+H^+} \text{Conjugate acid}$$

23.

H_3O^+ (acid), H_2O (conjugate base) and not OH^-.

24.

$$pH\,[HCl] = 2.0$$
$$\therefore \quad [H^+] = 10^{-2}\,M$$
$$[HCl] = 10^{-2}\,M$$
Volume = 200 mL
$$pH\,[NaOH] = 12.0$$
$$pOH = 2.0$$
$$[OH^-] = 10^{-2}\,M$$
$$[NaOH] = 10^{-2}\,M$$
Volume = 300 mL
$$N_1V_1\,(\text{acid}) = 200 \times 10^{-2} = 2$$
$$N_1V_2\,(\text{base}) = 300 \times 10^{-2} = 3$$
$$N_2V_2 > N_1V_1$$
Thus, resultant mixture basic.
$$N(OH^-) = \frac{N_2V_2 - N_1V_1}{V_1 + V_2} = \frac{3-2}{500} = 2\times10^{-3}\,M$$
$$pOH = -\log(2 \times 10^{-3}) = 2.7$$
$$\therefore \quad pH = 14 - pOH\ 14 - 2.7 = 11.3$$

25.

K_w changes with temperature. As temperature increases, $[OH^-]$ and $[H^+]$ decrease.

26.

Meq. of HCl = $10 \times 10^{-1} = 1$
Meq. of NaOH = $10 \times 10^{-1} = 1$
Thus both are neutralised and 1 Meq. of NaCl (a salt of strong acid and strong base) which does not hydrolyse and thus pH = 7.

27.

$$pK_w = -\log K_w = -\log 1 \times 10^{-12} = 12.$$
$$K_w = [H^+][OH^-] = 10^{-12}$$
$$[H^+] = [OH^-]$$
$$\Rightarrow \quad [H^+]^2 = 10^{-12};\ [H^+] = 10^{-6};\ pH = -\log[H^+] = -\log 10^{-6} = 6.$$
H_2O is neutral because $[H^+] = [OH^-]$ at 373 K even when pH = 6.
(d) is not correct at 373 K. Water cannot become acidic.

28.

$$AB_2(s) \rightleftharpoons \underset{s}{A^{2+}}(aq) + \underset{2s}{2B^-}(aq)$$

$$K_{sp} = [A^{2+}][B^-]^2 = (s)(2s)^2 = 4s^3$$
$$= 4(1.0 \times 10^{-5})^3 = 4 \times 10^{-15}$$

In the presence of 0.1 M A^{2+}, solubility is decreased due to common ion effect.

Let, solubility be = x mol L^{-1}

$$AB_2(s) \rightleftharpoons A^{2+}(aq) + 2B^-(aq)$$

A^{2+} (aq) added = 0.1 M

Total $[A^{2+}] = (x + 0.1\text{ M}) \approx 0.1$ M

$\because$ $x << 1.0 \times 10^{-5}$ M

$[B^-] = 2 \times$ M

$\therefore$ $[A^{2+}][B^-]^2 = 4 \times 10^{-15}$

$(0.1)(2x)^2 = 4 \times 10^{-15}$

$4x^2 = 4 \times 10^{-14}$

$x = 1 \times 10^{-7}$ M

29.

pH = 4 $\Rightarrow$ $[H^+] = 10^{-4}$ M $\Rightarrow$ $[OH^-] = 10^{-10}$ M

$$Al(OH)_3 \rightleftharpoons Al^{+3} + 3OH^-$$

K_{sp} $(Al(OH)_3) = [Al^{+3}][OH^-]^3$

$[Al^{3+}][OH^-]_3 = 1 \times 10^{-33}$

$[Al^{3+}](10^{-10})^3 = 1 \times 10^{-33}$ $\Rightarrow$ $[Al^{+3}] = 10^{-3}$ M

30.

$$K = 2 = \sqrt{k_1},\ K_2 = \frac{1}{K_4},\ K_1 = \frac{1}{K_3}$$

$\therefore$ $K_1K_3 = 1,\ \sqrt{K_1}\ K_4 = 1\ \sqrt{K_3}\ = 1$

31.
32.
33.
34.

35. The equilibrium constant of second reaction is very large and hence the equilibrium concentrations may be determined by adding the reactions. On adding,

	A	+	B	$\rightleftharpoons$	D	+	E	$K = K_1 \times K_2 = 1$
Initial moles	2		5		0		0	
Moles at equ.	$2 - x$		$5 - x$		x		x	
Equ. conc.	$\frac{2-x}{2}$M		$\frac{5-x}{2}$M		$\frac{x}{2}$M		$\frac{x}{2}$M	

Now, $$K = \frac{[D][E]}{[A][B]} = \frac{\left(\frac{x}{2}\right)\cdot\left(\frac{x}{3}\right)}{\left(\frac{2-x}{2}\right)\cdot\left(\frac{5-x}{2}\right)}$$

or, $$1 = \frac{x^2}{(2-x)(5-x)}$$

or $x = 1.428$

Now, for first reaction, $K_1 = \frac{[C][D]}{[A]}$

or, $$5 \times 10^{-6} = \frac{[C]\left(\frac{x}{2}\right)}{\left(\frac{2-x}{2}\right)}$$

$\therefore$ $[C] = \mathbf{2 \times 10^{-6}}$ **M**

36. **Example 39** *Some solid* NH_4HS *is introduced in a vessel containing* NH_3 *gas at* 0.5 atm. *Calculate the equilibrium partial pressures of gases. For the reaction:*
$NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$, $K_p = 0.12$ atm^2.

Solution

	$NH_4HS(g)$	$\rightleftharpoons$	$NH_3(g)$	+	$H_2S(g)$
Initial partial pressure			0.5 atm		0
Equ. par. pressure of gases			$(0.5 + x)$ atm		x atm

Now, $K_p = P_{NH_3} \cdot P_{H_2S}$

or, $0.12 = (0.5 + x) \times x$

or, $x = 0.177$

Hence, equilibrium pressure of $NH_3 = 0.5 + x = \mathbf{0.677\ atm}$

$H_2S = x = \mathbf{0.177\ atm}$

Now, $K = \frac{[D][E]}{[A][B]} = \frac{\left(\frac{x}{2}\right)\cdot\left(\frac{x}{3}\right)}{\left(\frac{2-x}{2}\right)\cdot\left(\frac{5-x}{2}\right)}$

or, $1 = \frac{x^2}{(2-x)(5-x)}$

or $x = 1.428$

Now, for first reaction, $K_1 = \frac{[C][D]}{[A]}$

or, $5 \times 10^{-6} = \frac{[C]\left(\frac{x}{2}\right)}{\left(\frac{2-x}{2}\right)}$

$\therefore$ $[C] = \mathbf{2 \times 10^{-6}\ M}$

37.

	$PCl_5(g)$	$\rightleftharpoons$	$PCl_3(g)$	+	$Cl_2(g)$
Initial moles	1 (say)		0		0
Moles at equ.	$1-\alpha$		α		α
Total moles at equ.	$= (1-\alpha) + \alpha + \alpha = 1 + \alpha$				
Equ. par pressure	$\frac{1-\alpha}{1+\alpha}\cdot P$		$\frac{\alpha}{1+\alpha}\cdot P$		$\frac{\alpha}{1+\alpha}\cdot P$

Now, $K_p = \frac{P_{PCl_3}\cdot P_{Cl_2}}{P_{PCl_5}} = \frac{\left(\frac{\alpha}{1+\alpha}\cdot P\right)\left(\frac{\alpha}{1+\alpha}\cdot P\right)}{\left(\frac{1-\alpha}{1+\alpha}\cdot P\right)} = \frac{\alpha^2 P}{1-\alpha^2}$

or, $0.2 = \frac{\alpha^2 \times 4}{1-\alpha^2}$

$\therefore$ $\alpha = \mathbf{0.218}$

[MATHEMATICS]

41. (c) **Trick :** $2a = 7$ or $a = \frac{7}{2}$

Also (5, –2) satisfies it, so $\frac{4}{49}(25) - \frac{51}{196}(4) = 1$

and $a^2 = \frac{49}{4} \Rightarrow a = \frac{7}{2}$.

42. (c) $(4x+8)^2 - (y-2)^2 = -44 + 64 - 4$

$\Rightarrow \frac{16(x+2)^2}{16} - \frac{(y-2)^2}{16} = 1$

Transverse and conjugate axes are $y = 2$, $x = -2$.

43. (c) Foci $(0, \pm 4) \equiv (0, \pm be) \Rightarrow be = 4$

Vertices $(0, \pm 2) \equiv (0, \pm b) \Rightarrow b = 2 \Rightarrow a = 2\sqrt{3}$

Hence equation is $\frac{-x^2}{(2\sqrt{3})^2} + \frac{y^2}{(2)^2} = 1$ or $\frac{y^2}{4} - \frac{x^2}{12} = 1$.

44. (b) Since $e > 1$ always for hyperbola and $\frac{2}{3} < 1$.

45. (a) The given equation is $2x^2 - 3y^2 = 5$

$\Rightarrow \frac{x^2}{5/2} - \frac{y^2}{5/3} = 1$

Now $b^2 = a^2(e^2 - 1) \Rightarrow \frac{5}{3} = \frac{5}{2}(e^2 - 1) \Rightarrow e = \sqrt{\frac{5}{3}}$.

The foci of hyperbola $(\pm ae, 0)$

$= \left(\pm\sqrt{\frac{5}{2}} \cdot \sqrt{\frac{5}{3}}, 0\right) = \left(\pm\frac{5}{\sqrt{6}}, 0\right)$.

46. (d) $ae = 1$, $a = 2$, $e = \frac{1}{2} \Rightarrow b = \sqrt{4\left(1 - \frac{1}{4}\right)} = \sqrt{3}$

Hence minor axis $= 2\sqrt{3}$.

47. (b) $\frac{x^2}{(48/3)} + \frac{y^2}{(48/4)} = 1$

$a^2 = 16, b^2 = 12 \Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2}$

Distance is $2ae = 2 \cdot 4 \cdot \frac{1}{2} = 4$.

48. (b) $\frac{x^2}{(30/2)} + \frac{y^2}{(30/3)} = 1 \Rightarrow \frac{x^2}{15} + \frac{y^2}{10} = 1$.

49. (c) $\frac{2b^2}{a} = 8,\ e = \frac{1}{\sqrt{2}} \Rightarrow a^2 = 64, b^2 = 32$

Hence required equation of ellipse is $\frac{x^2}{64} + \frac{y^2}{32} = 1$.

50. (d) Major axis = 3(Minor axis)

$\Rightarrow 2a = 3(2b) \Rightarrow a^2 = 9b^2 = 9a^2(1 - e^2) \Rightarrow e = \frac{2\sqrt{2}}{3}$

51. (b) $\frac{x^2}{2-r} + \frac{y^2}{r-5} + 1 = 0 \Rightarrow \frac{x^2}{r-2} + \frac{y^2}{5-r} = 1$

Hence $r > 2$ and $r < 5 \Rightarrow 2 < r < 5$.

52. (c) Given ellipse is $\frac{x^2}{\left(\frac{1}{3}\right)^2} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$

Here $b > a$; $\therefore$ Latus rectum $= \frac{2a^2}{b} = \frac{2 \times \frac{1}{9}}{\frac{1}{2}} = \frac{4}{9}$.

53. (c) Equation of the curve is $\frac{x^2}{5^2}+\frac{y^2}{4^2}=1$

$\Rightarrow -5 \le x \le 5,\ -4 \le y \le 4$

$PF_1+PF_2=\sqrt{[(x-3)^2+y^2]}+\sqrt{[(x+3)^2+y^2]}$

$=\sqrt{(x-3)^2+\frac{400-16x^2}{25}}+\sqrt{(x+3)^2+\frac{400-16x^2}{25}}$

$=\frac{1}{5}\left\{\sqrt{(9x^2+625-150x)}+\sqrt{(9x^2+625+150x)}\right\}$

$=\frac{1}{5}\left\{\sqrt{(3x-25)^2}+\sqrt{(3x+25)^2}\right\}=\frac{1}{5}\{25-3x+3x+25\}$

$=10$, ($\because 25-3x>0, 25+3x>0$)

54. (a) $\frac{x^2}{9}+\frac{y^2}{16}=(1+\sin 2t)+(1-\sin 2t)=2$.

55. (c) $E \equiv 4+9(3)^2-16(1)-54(3)+61<0$

Therefore, the point is inside the ellipse.

$\frac{4(x-2)^2}{36}+\frac{9(y-3)^2}{36}=1$

Equation of major axis is $y-3=0$ and point (1, 3) lies on it.

56. (c) Given equation of hyperbola, $\frac{x^2}{4}-\frac{y^2}{(16/9)}=1$,

$\therefore a=2,\ b=\frac{4}{3}$. As we know, $b^2=a^2(e^2-1)$

$\Rightarrow \frac{16}{9}=4(e^2-1) \Rightarrow e^2=\frac{13}{9}$, $\therefore e=\frac{\sqrt{13}}{3}$.

57. (c) The equation of hyperbola is $\frac{x^2}{16}-\frac{y^2}{9}=1$

Now $b^2=a^2(e^2-1) \Rightarrow e=\frac{5}{4}$

Hence foci are $(\pm ae, 0) \Rightarrow \left(\pm 4.\frac{5}{4}, 0\right)$ *i.e.*, $(\pm 5, 0)$.

58. (c) Using the condition the point (x_1, y_1) lies

(i) On the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$ if

$$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1=0$$

(ii) Outside the ellipse if $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1>0$

(iii) Inside the ellipse if $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1<0$

Given ellipse is $\frac{x^2}{1/4}+\frac{y^2}{1/5}=1$

$$\therefore \frac{16}{1/4}+\frac{9}{1/5}-1=64+45-1>0$$

Point (4, –3) lies outside the ellipse.

59. (a) $e=\sqrt{1+\frac{b^2}{a^2}} \Rightarrow e^2=\frac{a^2+b^2}{a^2}$

$$e_1=\sqrt{1+\frac{a^2}{b^2}} \Rightarrow e_1^2=\frac{b^2+a^2}{b^2} \Rightarrow \frac{1}{e_1^2}+\frac{1}{e^2}=1.$$

60. (d) For hyperbola $\Delta \neq 0$ and $h^2>ab$. Here $\Delta=0$.